Research Concerning the Kinetostatic Analysis of the Mechanism of the Conventional Sucker Rod Pumping Units

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It is well known that the bearings of the mechanism of the conventional pumping units are heavily loaded so that their design has to be accomplished very carefully. In this scope the values of the connection forces acting on these bearings has to be determined as accurately as possible. In the paper is presented the kinetostatic analysis of the mechanism of the conventional sucker rod pumping units, obtaining in this way the values of the connection forces in the joints and of the motor moment at the cranks shaft. For processing the experimental records it has been used the program Total Well Management. The simulations have been performed with a computer program developed by the authors using Maple programming environment.

Keywords: sucker rod pumping unit, kinetostatic analysis, connection forces

It is well known that the bearings of the conventional pumping units are some of their most loaded components, so for increasing safety in operation these bearings design has to be accomplished very carefully. For this purpose, the determination as accurate as possible of the values of the connection forces acting in the bearings plays an essential role, this being accomplished by an exact determination of the loads to which the component parts are subjected and by modeling the dynamics of the pumping unit mechanism. Some significant results regarding the kinematics and dynamics of the mechanism of the conventional pumping units that have strongly helped to the achievement of the research from this paper are presented in [1-5].

A method commonly used in the dynamics of the mechanisms for calculating the connection forces in the joints is that which uses the kinetostatic study [6-8]. In this paper is presented the kinetostatic analysis of the mechanism of the conventional sucker rod pumping units. The values of the force at the polished rod have been measured at a well serviced by a C-640D-305-120 pumping unit. The program *Total Well Management* [9] has been used for processing the experimental records. The simulations have been performed with a computer program developed by the authors using Maple programming environment [10].

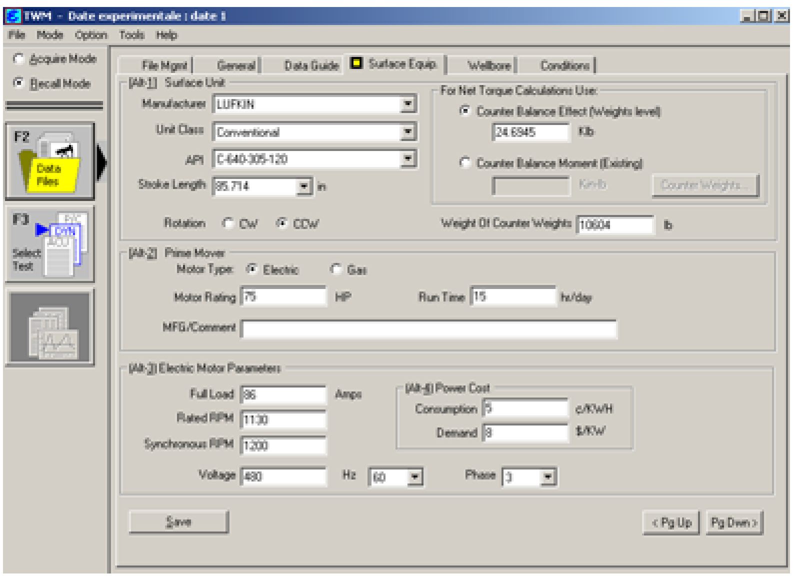

Fig. 1. Data on the surface equipment of the analyzed well

Experimental part

For processing the experimental records it has been used the program *Total Well Management* [10]. The analyzed well is serviced by a C-640D-305-120 pumping unit manufactured by *Lufkin* (fig. 1). The stroke length during operating is 85.714 in (2.177 m). The total weight of the four counterweights is of 10604 lbs (47169 N).

In establishing the variation on a cinematic cycle of the connection forces acting on the pumping unit mechanism bearings and of the motor torque at the crank shaft using the simulation program mentioned before were used the records concerning the variation of the force at the polished rod for two strokes: stroke 10 (fig. 2) and stroke 54 (fig. 3).

Kinetostatic analysis of the conventional sucker rod pumping units mechanism

The conventional pumping units mechanism is presented in figure 4. C_1, C_2 and C_3 are the mass centers of the cranks, connecting rods and of the rocker, respectively. m_{CG} represents the total mass of the balancing counterweights, m_{L1} is the total mass of the connecting bearings between the cranks and the connecting rods, m_{L2} is the mass of the spherical connecting bearing between the connecting rods and the rocker, m_{tr} is the mass of the equalizer traverse; m_{CB} is the mass of the rocker head considered to be concentrated in point *D'*, M_m is the motor

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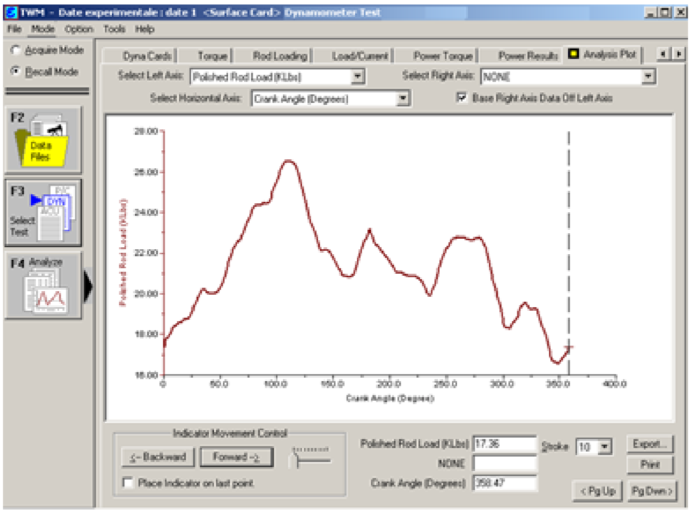


Fig. 2. The variation of the force at the polished rod during the stroke 10

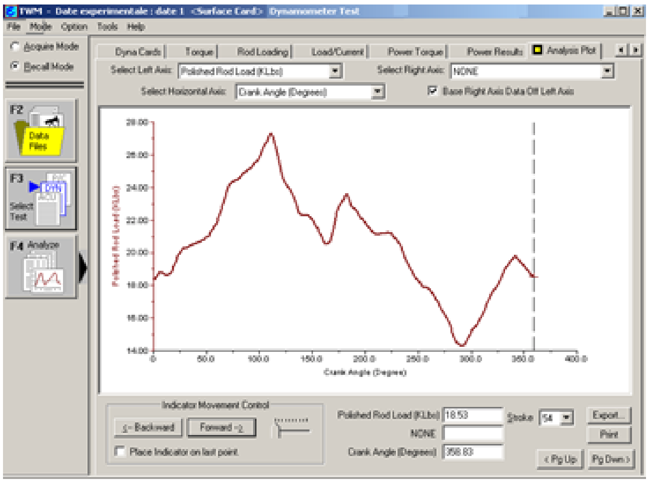


Fig. 3. The variation of the force at the polished rod during the stroke 54

moment at the cranks shaft; F is the force acting at the end of the polished rod.

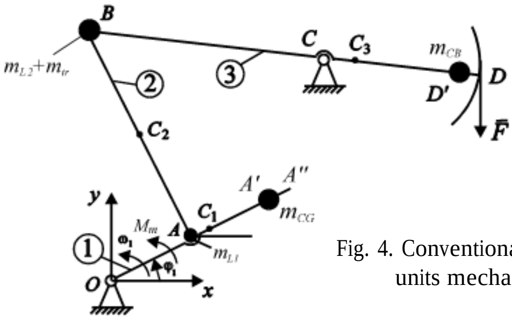
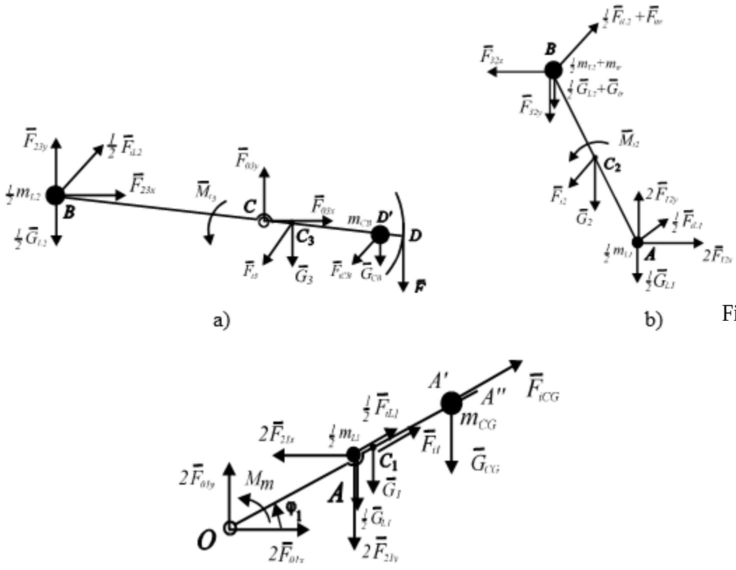


Fig. 4. Conventional pumping units mechanism

In figure 5 are presented the load schemes of the rocker, of the connecting rods and of the cranks. $G_j=m_{jj}\cdot g$; $j=\overline{1.3}$, are the weight forces of the cranks, of the connecting rods and of the rocker, respectively; m_1, m_2 and m_3 are their masses; $g=9.81$ m/s^2 is the gravitational acceleration that acts in the opposite direction of the *y* axis; $G_{CG}=m_{cG}\cdot g$ is the weight force of the balancing counterweights; $G_{L1}=m_{L1}\cdot g$ is the weight force of the two crank pin bearings (it has been considered that only half of the weight of these bearings is concentrated on the connecting rods, the rest remaining concentrated on the cranks); $G_{1,2}=m_{1,2}\cdot g$ is the weight force of the equalizer bearing; $G_{tr}=m_{tr}\cdot g$ is the weight force of the equalizer; $G_{CB}=m_{CB}\cdot g$ is the weight force of the horsehead; $\overline{F}_{ij}=-m_j\cdot\overline{a}_{Cj}$; $j=\overline{1.3}$ are the inertia forces corresponding to the cranks, connecting rods and to the rocker, respectively; $\overline{a}_{Cj}=\overline{1.3}$ are the accelerations of their mass centers; $M_{ij}=-J_{Cj}\cdot\overline{\varepsilon}_j$, $j=\overline{2.3}$ are the inertia moments, where J_{Cj}, $j=\overline{2.3}$, are the mass moments of inertia corresponding to the connecting rods and to the rocker and $\overline{\varepsilon}_j$; $j=\overline{2.3}$, are their angular accelerations; $\overline{F}_{iCG}=-m_{CG}\cdot\overline{a}_{A'}$; $\overline{F}_{iL1}=-m_{L1}\cdot\overline{a}_A$, $\overline{F}_{iL2}=-m_{L2}\cdot\overline{a}_B$, $\overline{F}_{itr}=-m_{tr}\cdot\overline{a}_B$ and $\overline{F}_{iCB}=-m_{CB}\cdot\overline{a}_{D'}$ are the inertia forces corresponding to the balancing counterweights, to the two crank pin bearings, equalizer bearing, equalizer and to the horsehead, respectively.

The manner of determining the coordinates of different points on the mechanism where acting forces and moments, the speeds and the accelerations of these points and the angular speeds and accelerations of the connection rods and of the rocker depending on the dimensions of the component elements of the pumping unit, the crank angle φ_{1d} and the angular speed of the cranks ω_1 is presented in [4, 5]. In [5] it is also presented the manner of determining the surface stroke of the pumping unit and of the crank angles φ_{1d} and φ_{1a} corresponding to the beginning of the upward and downward movements of the sucker rod column.

In figure 5 are represented the projections on *x* and *y* axes of the connection force from the centre bearing ($\overline{F}_{03x}$ and $\overline{F}_{03y}$); the projections on *x* and *y* axes of the



a)

b)

c)

Fig. 5. Load schemes of the rocker (a), of the connecting rods (b) and of the cranks (c)

connection force $\overline{F}_{23}$ ($\overline{F}_{23x}$ and $\overline{F}_{23y}$) from the equalizer bearing acting on the rocker; the projections on x and y axes of the connection force $\overline{F}_{32}$ ($\overline{F}_{32x}=-\overline{F}_{23x}$ and $\overline{F}_{32y}=-\overline{F}_{23y}$) from the equalizer bearing acting on the equalizer; the projections on x and y axes of the connection forces from the two crank pin bearings($2\overline{F}_{12x}$ and $2\overline{F}_{12y}$) acting on the connecting rods; the projections on x and y axes of the connection forces from the two crank pin bearings ($2\overline{F}_{21x}=-2\overline{F}_{12x}$ and $2\overline{F}_{21y}=-2\overline{F}_{12y}$) acting on the cranks; the projections on x and y axes of the connection forces from the two joints connecting the two cranks and the output shaft of the reducer ($2\overline{F}_{01x}$ and $2\overline{F}_{01y}$).

In relations (1), (2) and (3) are given the dynamic equilibrium equations in forces and moments corresponding to the rocker, connecting rods and cranks, respectively.

$$\begin{cases}\left(\sum F_x\right)_3=0\Rightarrow F_{23x}+F_{03x}+\frac{1}{2}F_{iL2x}+F_{i3x}+F_{iCBx}=0\\ \left(\sum F_y\right)_3=0\Rightarrow F_{23y}+F_{03y}+\frac{1}{2}F_{iL2y}+F_{i3y}+F_{iCBy}-\frac{1}{2}G_{L2}-G_3-G_{CB}-F=0\\ \left(\sum M_C\right)_3=0\Rightarrow \overline{CB}\times(\overline{F}_{23x}+\overline{F}_{23y})+\overline{CB}\times(\frac{1}{2}\overline{F}_{iL2}+\frac{1}{2}\overline{G}_{L2})+\overline{CC_3}\times(\overline{F}_{i3}+\overline{G}_3)+\\ \qquad+\overline{CD}'\times(\overline{F}_{iCB}+\overline{G}_{CB})+\overline{CD}\times\overline{F}+\overline{M}_{i3}=0\end{cases}\tag{1}$$

$$\begin{cases}\left(\sum F_x\right)_2=0\Rightarrow F_{32x}+2F_{12x}+\frac{1}{2}F_{iL2x}+F_{itrx}+F_{i2x}+\frac{1}{2}F_{iL1x}=0\\ \left(\sum F_y\right)_2=0\Rightarrow F_{32y}+2F_{12y}+\frac{1}{2}F_{iL2y}+F_{itry}+F_{i2y}+\frac{1}{2}F_{iL1y}-\frac{1}{2}G_{L2}-G_{tr}-G_2-\frac{1}{2}G_{L1}=0\\ \left(\sum M_B\right)_2=0\Rightarrow \overline{BA}\times(2\overline{F}_{12x}+2\overline{F}_{12y})+\overline{BA}\times(\frac{1}{2}\overline{F}_{iL1}+\frac{1}{2}\overline{G}_{L1})+\overline{BC_2}\times(\overline{F}_{i2}+\overline{G}_2)+\overline{M}_{i2}=0\end{cases}\tag{2}$$

$$\begin{cases}\left(\sum F_x\right)_1=0\Rightarrow 2F_{01x}+2F_{21x}+\frac{1}{2}F_{iL1x}+F_{i1x}+F_{iCGx}=0\\ \left(\sum F_y\right)_1=0\Rightarrow 2F_{01y}+2F_{21y}+\frac{1}{2}F_{iL1y}+F_{i1y}+F_{iCGy}-\frac{1}{2}G_{L1}-G_1-G_{CG}=0\\ \left(\sum M_A\right)_1=0\Rightarrow \overline{AO}\times(2\overline{F}_{01x}+2\overline{F}_{01y})+\overline{M}_m+\overline{AC_1}\times\overline{G}_1+\overline{AA'}\times\overline{G}_{GC}=0\end{cases}\tag{3}$$

The dynamic equilibrium equations from relations $1\div3$ have been transposed into a compact matrix form:

$$A\cdot X=B\tag{4}$$

where:

$$X=\begin{bmatrix}F_{01x} & F_{01y} & M_m & F_{12x} & F_{12y} & F_{23x} & F_{23y} & F_{03x} & F_{03y}\end{bmatrix}^T\tag{5}$$

Then, the projections on x and y axes of the connection forces $\overline{F}_{01},\overline{F}_{12},\overline{F}_{23},\overline{F}_{03}$ and the motor moment M_m at the cranks shaft may be determined with the relation:

$$X=A^{-1}\cdot B\tag{6}$$

In relation (7) is presented the expression of the matrix A and the elements of the vector B are given in relations $8\div16$:

$$A=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & -(y_B-y_C) & x_B-x_C & 0 & 0\\ 0 & 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 2 & 0 & -1 & 0 & 0\\ 0 & 0 & 0 & -2(y_A-y_B) & 2(x_A-x_B) & 0 & 0 & 0 & 0\\ 2 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & -2 & 0 & 0 & 0 & 0\\ 2y_A & -2x_A & 1 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\tag{7}$$

$$B_1=-\frac{1}{2}F_{iL2x}-F_{i3x}-F_{iCBx}\tag{8}$$

$$B_2=-\frac{1}{2}F_{iL2y}-F_{i3y}-F_{iCBy}+\frac{1}{2}G_{L2}+G_3+G_{CB}+F\tag{9}$$

$$\begin{aligned}B_3=&-\frac{1}{2}[(F_{iL2y}-G_{L2})\cdot(x_B-x_C)-F_{iL2x}\cdot(y_B-y_C)]-[(F_{i3y}-G_3)\cdot(x_{C_3}-x_C)-\\ &-F_{i3x}\cdot(y_{C_3}-y_C)]-[(F_{iCBy}-G_{CB})\cdot(x_{D'}-x_C)-F_{iCBx}\cdot(y_{D'}-y_C)]+F\cdot(x_D-x_C)+J_{C_3}\cdot\varepsilon_3\end{aligned}\tag{10}$$

$$B_4=-\frac{1}{2}F_{iL2x}-F_{itrx}-F_{i2x}-\frac{1}{2}F_{iL1x}\tag{11}$$

$$B_5 = -\frac{1}{2}F_{iL2y} - F_{itry} - F_{i2y} - \frac{1}{2}F_{iL1y} + \frac{1}{2}G_{L2} + G_{tr} + G_2 + \frac{1}{2}G_{L1} \tag{12}$$

$$B_6 = -\frac{1}{2}[(F_{iL1y} - G_{L1})\cdot(x_A - x_B) - F_{iL1x}\cdot(y_A - y_B)] - \\ -[(F_{i2y} - G_2)\cdot(x_{C_2} - x_B) - F_{i2x}\cdot(y_{C_2} - y_B)] + J_{C_2}\cdot\varepsilon_2 \tag{13}$$

$$B_7 = -\frac{1}{2}F_{iL1x} - F_{i1x} - F_{iCGx} \tag{14}$$

$$B_8 = -\frac{1}{2}F_{iL1y} - F_{i1y} - F_{iCGy} + \frac{1}{2}G_{L1} + G_1 + G_{CG} \tag{15}$$

$$B_9 = G_1\cdot(x_{C_1} - x_A) + G_{GC}\cdot(x_{A'} - x_A) \tag{16}$$

Simulation results and discussions

The kinetostatic analysis of the conventional sucker rod pumping units mechanism presented before has been transposed by the authors into a computer program using Maple programming environment [10]. The simulations have been performed in the case of a C-640D-305-120 pumping unit produced by *Lufkin* [11], whose component elements have the following dimensions: *OA* = 30 in. (0.762 m); *AB* = 133.5 in. (3.3909 m); *BC* = 111.09 in. (2.8217 m); *CD* = 155 in. (3.937 m). The coordinates of the point *C* (fig. 4) are [10]: $x_C = 111$ in. (2.8194 m) and $y_C = 138$ in. (3.5052 m). The values of the crank angles φ_{1d} and φ_{1a} are: 88.976° and 266.929° , respectively.

The simulations have been accomplished by considering the following values of the other parameters involved in calculations: *CD'* = 140 in. (3.556 m); *OA'* = 54.5 in. (1.3843 m); m_{L1} = 88 kg; m_{L2} = 169 kg; m_{tr} = 580 kg; m_{CB} = 840 kg; q_1 = 722 kg/m; q_2 = 34 kg/m; q_3 = 300 kg/m (q_1, q_2 and q_3 are the linear masses of the cranks, connecting rods and of the rocker, respectively). The work angular speed of the cranks has been of 6.6 rot/min.

In figures 6÷10 are presented the variation on a cinematic cycle of the values of the connection forces $F_{01}, F_{12}, F_{23}, F_{03}$ and of the motor moment M_m at the cranks shaft for the strokes 10 (curves *1*) and 54 (curves *2*).

Figures 6÷9 highlight the extremely high values of the connection forces acting in the bearings of the analyzed pumping unit, especially the values of the connection force F_{03}. The differences occurring when the crank angle φ_1 varies between 250 and 320 degrees between the calculated values of the connection forces for the two strokes 10 and 54 are mainly due to the differences occurring for the two strokes regarding the variation of the force *F* at the polished rod in this angular range (figs. 3 and 4). Regarding the variation of the motor moment M_m (fig. 10), it has the same allure for the two strokes 10 and 54, differences appearing in the same range of variation of the angle φ_1 (250-320 degrees) due to the variation of the force *F* at the polished rod in this angular range.

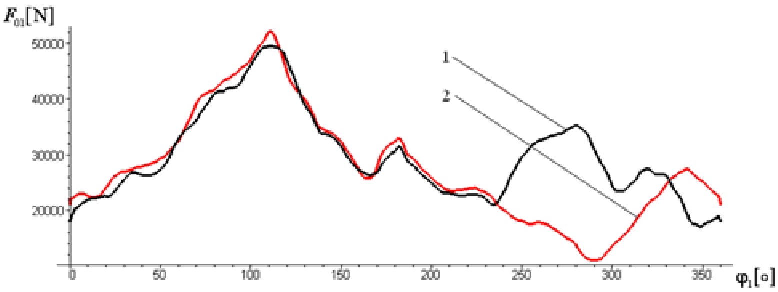


Fig. 6. The variation on a cinematic cycle of the connection force F_{01} for the stroke 10 (curve *1*) and the stroke 54 (curve *2*)

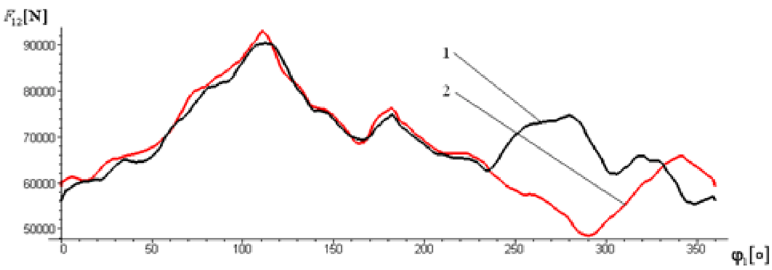


Fig. 7. The variation on a cinematic cycle of the connection force F_{12} for the stroke 10 (curve *1*) and the stroke 54 (curve *2*)

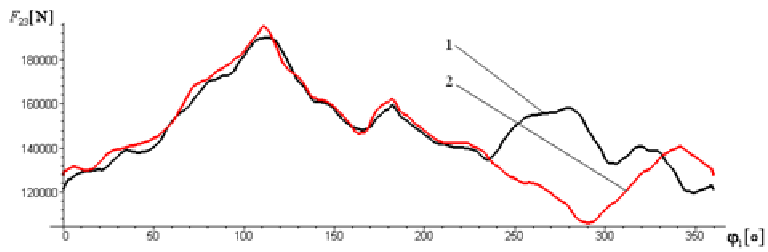


Fig. 8. The variation on a cinematic cycle of the connection force F_{23} for the stroke 10 (curve *1*) and the stroke 54 (curve *2*)

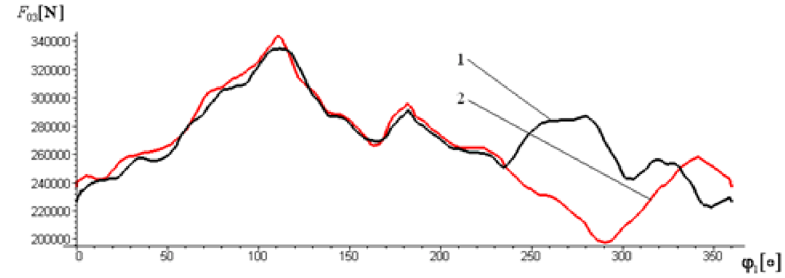


Fig. 9. The variation on a cinematic cycle of the connection force F_{03} for the stroke 10 (curve *1*) and the stroke 54 (curve *2*)

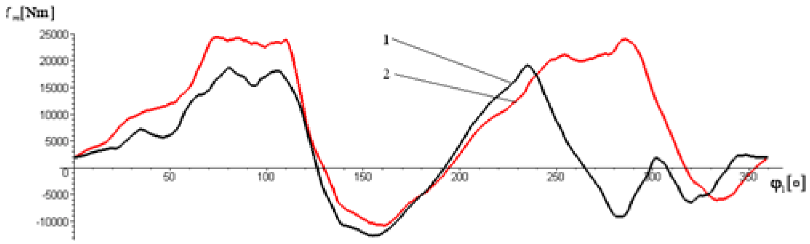


Fig. 10. The variation on a cinematic cycle of the motor moment M_m for the stroke 10 (curve *1*) and the stroke 54 (curve *2*)

Conclusions

In this paper it has been presented the kinetostatic analysis of the mechanism of the conventional sucker rod pumping units, highlighting the manner of determining the connection forces in bearings and the motor torque at the crank shaft. The program *Total Well Management* has been used for processing the experimental records and the simulations have been performed with a computer program developed by the authors using Maple programming environment. The results obtained after simulations in the case of a C-640D-305-120 pumping unit underlined the extremely high values of the connection forces acting in the bearings of the analyzed pumping unit, especially the values of the connection force acting in the centre bearing. Therefore one of the directions of future research is to develop a method of optimizing the mechanism of conventional pumping units having as one of the objectives the reduction of the extreme values of the connecting forces in the component bearings.

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